which, of course, is the authors' result [Eq. (A15)], because $\Omega_n \approx 0$ is equivalent to $I_{22} \approx I_{33}$. However, returning to Eq. (2), with $\dot{h} = 0$ and using (4) as the initial conditions, we have

$$\omega(t) = (-h/I_{11} + khp/I_{22}\Omega_n)e^{-i\Omega_n t} - khp/I_{22}\Omega_n \qquad (5)$$

which expands to

$$\omega_1(t) = h\left(\frac{kp}{I_{22}\Omega_n} - \frac{1}{I_{11}}\right)\cos\Omega_n t - \frac{khp}{I_{22}\Omega_n} \qquad (6)$$

$$\omega_2(t) = h(1/kI_{11} - p/I_{22}\Omega_n)\sin\Omega_n t \qquad (7)$$

The system performance should now be evaluated on the basis of Eqs. (6) and (7) with $k \to 0$, $\Omega_n \to 0$, rather than the authors' Eqs. (A18) and (A21). Using the Euler angles of Fig. 1 to define the inertial motion of the body axes, the angles of interest (for small angles) are given by

$$\dot{\alpha} \approx \omega_1 + \beta p \qquad \dot{\beta} \approx \omega_2 - \alpha p \qquad (8)$$

Substituting (6) and (7) in Eqs. (8) with $\alpha(0) = \beta(0) = 0$ and $\dot{\alpha}(0) = -h/I_{11}$, the solution for the tilt angle α and the cant angle β is

$$\alpha = -h(\sin\Omega_n t)/I_{11}\Omega_n \qquad (9)$$

$$\beta = h(1 - \cos\Omega_n t)/p(I_{11} - I_{33}) \qquad (10)$$

Equations (9) and (10) define the deviation of the e_3 axis from the angular momentum vector. In the limit as $\Omega_n \to 0$ (i.e., $I_{22} \to I_{33}$), Eq. (9) reduces to the authors' result $\alpha = -ht/I_{11}$ as given by Eq. (A23). But Table 1 clearly shows that the concept cannot be reduced to practice. The authors' values are used, with $\omega_0 = p = 10.46$ rad/sec, $J_0\Omega = h = 0.0225$ ft-lb-sec, and $I_{11} = 124$ slug-ft^2. I_{33}/I_{11} is taken to be 1.4.

It is clear that the e_3 axis never departs very far from the angular momentum vector even when I_{22} and I_{33} differ by less than one part in a million. The requirements on I_{22} and I_{33} for α to reach the desired 15° are

$$1 \geq I_{22}/I_{33} > 0.99999999 \text{ (approximately)} \qquad (11)$$

Also, the cant angle β is not zero and must be accounted for. For example, $\beta_{max} = 2h/[p(I_{33} - I_{11})]$, which for the preceding example is 0.005°, a value probably unacceptable for the 2400-line, 15° picture.

There are some other areas of difficulty in the proposed technique (i.e., alignment, thermal distortion, and residual excess energy). Since there is no preferred spin axis, any finite amount of excess energy in the system will result in an unwanted drift about the e_1 axis. The difficulty inherent in stabilizing a spinning body having two nearly equal moments of inertia is analyzed on an energy dissipation basis in Ref. 3.

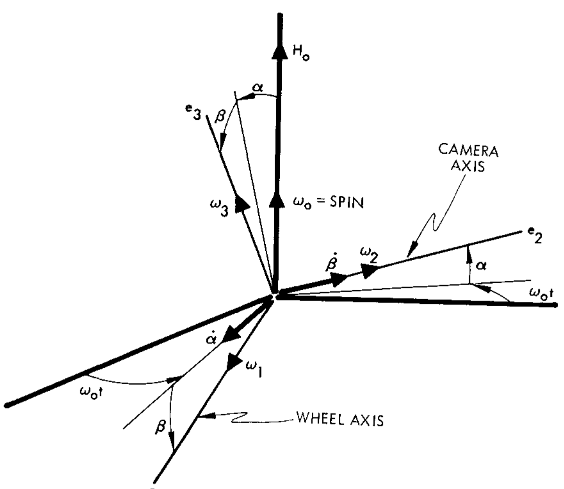

Fig. 1 Euler angles defining spacecraft motion.

Table 1 Maximum available tilt angle, α_{max}

I_{22}/I_{33}	Ω_n, rad/sec	α_{max}, deg
0.99	0.66	0.0158
0.999	0.209	0.0498
0.9999	0.066	0.158
0.99999	0.0209	0.498
0.999999	0.0066	1.58

In summary, it may be said that the "practical method of control" referred to in the authors' footnote[1] must be directed toward the inequality (11) which leaves little for the engineer to work with when considering the proposed technique.

References

[1] Beachley, N. H. and Uicker, J. J., Jr., "Wobble-Spin Technique for Spacecraft Inversion and Earth Photography," *Journal of Spacecraft and Rockets*, Vol. 6, No. 3, March 1969, pp. 245–248.

[2] Grasshoff, L. H., "An Onboard, Closed-Loop Nutation Control System for a Spin-Stabilized Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 5, No. 5, May 1968, pp. 530–535.

[3] Kuebler, M. E., "Gyroscopic Motion of an Unsymmetrical Satellite Under No External Forces," TN D-596, Dec. 1960, NASA.

Reply by Authors to L. H. Grasshoff

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THE point raised by L. H. Grasshoff is very significant. His analysis shows the extreme mechanical accuracy necessary for the example of Ref. 1 to work in the manner indicated, and should contribute to a better understanding of systems of this type. Our computer simulation results, from continuing development of this concept, agree with the values presented by Grasshoff. We must agree, therefore, that the concept in its pure form is indeed very sensitive to small errors in the I_{22}/I_{33} ratio (nominally unity), much more so than one would like.

The numbers presented by Grasshoff in his Table 1 all apply to the "stable" case, with $I_{33} > I_{22}$. It is equally important, in working out the details of a practical system, to consider the "unstable" case with $I_{33} < I_{22}$. (Since we also have $I_{33} > I_{11}$, this corresponds to the condition of "unstable equilibrium" of a rigid body rotating about its principal axis of intermediate moment of inertia.) With this unstable condition, the tilt rate $\dot{\alpha}$ will increase as the tilt angle α increases (up to 90°), the amount of this increase depending upon degree of instability. We will discuss individually the two applications: a) inversion of a spin-stabilized satellite, and b) spin-scan earth photography with the camera rigidly attached to the basic satellite structure.

Spacecraft Inversion

For inversion of a spin-stabilized satellite, one probably is not concerned with the exact tilt rate or the degree of nutation during the maneuver itself, but merely wants a quick, reliable, low-energy-consumption technique. A larger flywheel could overcome a practical error tolerance in the I_{22}/I_{33} ratio and reduce inversion time, but the flywheel's size and power consumption would no longer be insignificant. Probably a better approach is to have the spacecraft slightly stable during normal operation, but changed (as by a small translating

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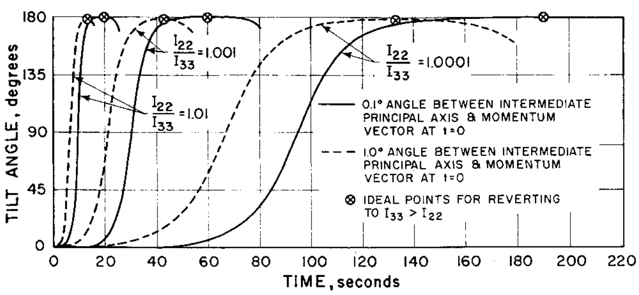

Fig. 1 Inversion maneuver of system without flywheel, controlled by weight translation alone.

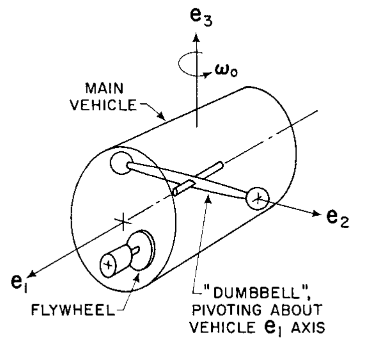

Fig. 2 The dumbbell variation of the wobble-spin system.

weight) to be slightly unstable during the inversion maneuver. If this is done, the maneuver can be shown to work very nicely. The exact degree of instability is not too important, but the speed of the maneuver increases with the degree of instability. But even if made to be highly unstable during the maneuver, the satellite is still in equilibrium (even though it is unstable equilibrium) at its original attitude and the completely inverted attitude, so that $\dot{\alpha}$ is very low around these two positions. The exact timing of the translating weight is therefore not too critical.

Since nutation is not a concern, it is also feasible to omit the flywheel for this inversion maneuver, accomplishing it by weight translation alone. Figure 1 shows how an inversion maneuver would take place for three values of I_{22}/I_{33}. With no flywheel, inversion would theoretically occur only if there were an offset angle ϕ between the spin axis (angular momentum vector) and the principal axis of intermediate moment of inertia at the beginning of the maneuver. In practice, of course, it would be impossible to achieve $\phi = 0$, but the question might be raised as to whether or not one should design the weight movements to achieve a certain angular offset ϕ_0 at the start. In Fig. 1, curves for $\phi_0 = 1.0°$ and 0.1° (the angles being in the e_2e_3 plane), have been plotted for the three I_{22}/I_{33} ratios. Although the maximum $\dot{\alpha}$ is virtually unaffected by ϕ_0, there are two points in favor of using a very small angle: 1) The α achieved before the satellite starts to return to its initial attitude is greater. With $\phi_0 =$ 1.0°, for example, a tilt of 178° is achieved, whereas a 0.1° offset gives 179.8°. The closer to a full 180° one can achieve, the smaller the oscillations that have to be eliminated after "recapture" (i.e., returning to the condition of $I_{33} > I_{22}$). 2) The smaller the ϕ_0, the greater the period of time, for any given I_{22}/I_{33} ratio, during which satisfactory recapture can be achieved. It might be advantageous to use a relatively high ϕ_0, however, if one were to perform the maneuver by a programed weight movement, i.e., by maintaining the $I_{22} > I_{33}$ condition for a predetermined period of time. The smaller the initial offset, the closer the tolerance one would have to hold on its exact value to get an accurate estimate of the time required for the maneuver.

It might be emphasized here that such inversion could be very valuable for synchronous satellites even without devices like the radiation cooler mentioned in Ref. 1. It would be convenient to keep the same end of the satellite in the shade throughout the entire year, since less extreme variations in thermal environment would facilitate the design of all exposed satellite components.

Spin-Scan Photography with Fixed Camera

Here the high sensitivity pointed out by Grasshoff presents a greater challenge, as a precise $\dot{\alpha}$ is imperative, and excessive nutation cannot be tolerated. Even if the I_{22}/I_{33} ratio could be adjusted on the ground to an accuracy of better than one part in 10^8, it would be unrealistic to expect it to maintain this accuracy during operation, with thermal expansion, centrifugal forces, and nonuniform compliance.

A feedback control system could be used to make adjustments to the I_{22}/I_{33} ratio during the course of the maneuver, based on the actual tilting action. With such a system, the satellite would alternate between the stable and unstable condition. Preliminary studies indicate that such a control system would work very well, but it would have the disadvantage of requiring a signal of either α or $\dot{\alpha}$ to an accuracy somewhat better than that to be maintained.

A slight modification of the basic wobble-spin concept has evolved that looks promising as a technique to be operated in an open-loop manner. This modified system (Fig. 2) consists of the basic spacecraft, a small flywheel, and a pair of weights, or "dumbbell," located along the intersection of the e_2e_3 plane and a plane perpendicular to the spin axis. To give the desired $\dot{\alpha}$, the flywheel is rotated exactly as in the basic concept while the dumbbell is rotated, with respect to the main body, in the same direction as the flywheel but at a speed equal in value, although opposite in direction, to $\dot{\alpha}$. The two dumbbell weights will remain therefore in a plane perpendicular to the instantaneous spin axis throughout the entire tilting maneuver. The dumbbell would probably be mechanically geared to the flywheel. If the combination of the basic vehicle and flywheel (i.e., the total satellite except for the dumbbell) is made "cylindrical" about the e_1 axis, then this system will allow the instantaneous spin axis to remain the axis of maximum moment of inertia throughout the maneuver. This system could be used for inversion too, of course, if the dumbbell were given freedom to rotate at least 180°.

It is relatively easy to analyze a perfect system of this type. The equation for tilt rate is, with clarification of I_{11}, the same as for the basic concept,[1]

$$\dot{\alpha} = -J_0\Omega/I_{11}$$

where J_0 is the moment of inertia of the flywheel, Ω is the flywheel speed relative to main body, and I_{11} is the moment of inertia, about the e_1 axis, of the total vehicle excluding the dumbbell.

An investigation is under way to determine its sensitivity to various combinations of mechanical errors. If the dumbbell is large enough, then the system will be relatively insensitive to mechanical errors, as may be seen by visualizing an extreme case: a satellite in which the nontilting dumbbell comprises 99.9% of its inertia, the tilting "main body" is a tiny cylinder, and the flywheel is almost infinitesimal. One goal of current work is to answer the following question: With the type of mechanical accuracy one can maintain in practice with such a satellite, what is the size dumbbell necessary for satisfactory open-loop spin-scan operation? The attractiveness of this modification depends, of course, on the answer.

In summary, the basic wobble-spin concept—that a "cylinder" spin-stabilized about a transverse axis can be tilted about its axis of symmetry by internal mass movement alone—is sound. There are problems to be worked out, but further development and some ingenuity will, it is felt, lead to practical systems that are very useful for certain types of satellites.

Reference

[1] Beachley, N. H. and Uicker, J. J., Jr., "Wobble-Spin Technique for Spacecraft Inversion and Earth Photography," *Journal of Spacecraft and Rockets*, Vol. 6, No. 3, March 1969, pp. 245–248.